O-minimality and Diophantine Geometry
September 26-30, 2022
L3

Abstracts

Gal Binyamini (Weizmann Institute)

Title: Sharp o-minimality

Abstract: I'll recall the notion of sharp o-minimality that we recently introduced to study Wilkie's conjecture and related arithmetic properties of definable sets and the main results in this direction. I'll also discuss a recent work with Novikov and Zack laying out some foundational properties of sharply o-minimal structures, and a work in progress with Gabrielov an Vorobjov where we show how to go from "restricted" sharp structures (reducts of R_an) to "unrestricted" ones (such as R_exp).

Laura Capuano (University of Roma Tre)

Title: GCD results on semiabelian varieties and a conjecture of Silverman

Abstract: A divisibility sequence is a sequence of integers d_n such that, if m divides n, then d_m divides d_n. Bugeaud, Corvaja, Zannier showed that pairs of divisibility sequences of the form a^n-1 have only limited common factors. From a geometric point of view, a divisibility sequence corresponds to a subgroup of the multiplicative group, and Silverman conjectured that a similar behavior should appear in (a large class of) other algebraic groups.

Extending previous works of Silverman and of Ghioca-Hsia-Tucker on elliptic curves over function fields, we will show how to prove the analogue of Silverman's conjecture over function fields in the case of abelian and split semiabelian varieties and some generalizations. The proof relies on some results of unlikely intersections. This is a joint work with F. Barroero and A. Turchet.

Gabriel Dill (University of Hannover)

Title: Arithmetic unlikely intersections in split semiabelian varieties

Abstract: In joint work in progress with Francesco Campagna that is inspired by a result of Bugeaud, Corvaja, and Zannier, we formulate a conjecture about unlikely intersections in split semiabelian schemes over the spectrum of some ring of S-integers in a number field. This involves the definition of a complexity on the set of endomorphisms of a split semiabelian variety that might be of interest in itself. I will show some evidence that we have acquired so far for our conjecture and discuss in some detail the example of a certain arithmetic surface inside the fibered cube of the multiplicative group over the integers.

Ziyang Gao (University of Hannover)

Title: Torsion points in families of abelian varieties

Abstract: Given an abelian scheme defined over \IQbar and an irreducible subvariety X which dominates the base, the Relative Manin-Mumford Conjecture (inspired by S.Zhang and proposed by Zannier) predicts how torsion points in closed fibers lie on X. The conjecture says that if such torsion points are Zariski dense in X, then the dimension of X is at least the relative dimension of the abelian scheme, unless X is contained in a proper subgroup scheme. In this talk, I will present a proof of this conjecture. As a consequence this gives a new proof of the Uniform Manin-Mumford Conjecture for curves (recently proved by Kühne) without using equidistribution. This is joint work with Philipp Habegger.

Philipp Habegger (University of Basel)

Title: Some cases of the Schinzel-Zassenhaus Conjecture in Arithmetic Dynamics

Abstract: In this joint work with Harry Schmidt we prove a dynamical variant of the Schinzel-Zassenhaus Conjecture for a class of polynomials that contains T^2-1. For this polynomial we obtain a lower bound for the Call-Silverman height of a wandering point that decays like the inverse of the square of the field degree. Our method is based on a recent breakthrough by Vesselin Dimitrov who proved the Schinzel-Zassenhaus Conjecture.

Annette Huber (University of Freiburg)

Title: Exponential periods and o-minimality

Abstract: (joint work with J. Commelin and P. Habegger) Periods are complex numbers obtained by integrating an algebraic differential from defined over the rationals over some domain also of algebraic nature. This includes π and $log(\alpha)$ for algebraic numbers α. They are the object of long-standing conjectures in transcendence theory. In recent years, a variant has come into focus that also allows for factors $\exp(f)$ where f is an algebraic function.

We explain how this is related to a certain o-minimal structure and speculate on applications to the theory of periods.

David Masser (University of Basel)

Title: Some new elliptic integrals

Abstract: In 1981 James Davenport surmised that if an algebraic function $f(x,t)$ is not integrable (with respect to x) by elementary means when t is an independent variable, then there are most finitely many complex numbers τ such that $f(x,\tau)$ is integrable by elementary means. In 2020 Umberto Zannier and I obtained a couple of counterexamples and in principle classified all of them with algebraic coefficients (they are somewhat rare). In this talk I will review our work and sketch our recent practical description of all elliptic counterexamples; they are not unrelated to Ribet curves.

Alina Ostafe (University of New South Wales)

Title: Integer matrices with a given characteristic polynomial and multiplicative dependence of matrices

Abstract: We consider the set $\mathcal{M}_n(\mathbb{Z}; H))$ of $n\times n$-matrices with integer elements of size at most H and obtain upper and lower bounds on the number of s-tuples of matrices from $\mathcal{M}_n(\mathbb{Z}; H)$, satisfying various multiplicative relations, including multiplicative dependence, commutativity and bounded generation of a subgroup of $\mathrm{GL}_n(\mathbb{Q})$. These problems generalise those studied in the scalar case $n=1$ by F. Pappalardi, M. Sha, I. E. Shparlinski and C. L. Stewart (2018) with an obvious distinction due to the non-commutativity of matrices. As a part of our method, we obtain a new upper bound on the number of matrices from $\mathcal{M}_n(\mathbb{Z}; H)$ with a given characteristic polynomial $f \in\mathbb{Z}[X]$, which is uniform with respect to f. This complements the asymptotic formula of A. Eskin, S. Mozes and N. Shah (1996) in which f has to be fixed and irreducible.

Joint work with Igor Shparlinski.

Kobi Peterzil (University Haifa)

Title: The Housdorff limit of images of definable sets in the torus (j. with Sergei Starchenko)

Abstract: Let L be a lattice in R^n, T the torus R^n/L and p: R^n -> T.

In a previous work we considered o-minimally defined subsets X in R^n
and described the closure of p(X) in T, uniformly in L, in terms of certain affine sets in the reals, associated to types on X, over the reals.

We now consider a definable family of sets {X_t: t in R} in R^n, and describe the possible Huasdorff limits of p(X_t) in T, as t tends to +infinity. We do that (again uniformly in L) using affine sets associated to types on X_t, for nonstandard t.

The problem is a topological analogue to measure theoretic problems in Ergodic theory, regarding so-called dilations on tori and nilmanifolds.

Marta Pieropan (Utrecht University)

Title: The split torsor method for Manin's conjecture

Abstract: A conjecture of Manin predicts an asymptotic formula for the number of rational points of bounded anticanonical height on Fano varieties over number fields. The split torsor method consists in parameterizing the rational points by lattice points on a suitable split torsor. Over arbitrary number fields, the lattice point counting is carried out in the framework of o-minimal structures. I will report on joint work with U. Derenthal where we prove Manin's conjecture for all nonsplit quartic del Pezzo surfaces of type A3+A1 over arbitrary number fields.

Jonathan Pila (University of Oxford)

Title: Primitives of algebraic functions

Abstract: In joint work with Jacob Tsimerman we study when the primitive of a given algebraic function can be constructed using primitives from some given finite set of algebraic functions, their inverses, algebraic functions, and composition. When the given finite set is just {1/x} this is the classical problem of "elementary integrability". I will discuss some results, including a decision procedure for this question, and further problems and conjectures.

Tom Scanlon (University of California, Berkeley)

Title: Likely intersections

Abstract: The Zilber-Pink conjectures predict that if X is an irrediucible variety in some special variety S but is not contained in a proper special subvariety of S, then the union of the unlikely intersections of X with special subvarieties S' of S, meaning those for which dim(X) + dim(S') <

dim(S), is not Zariski dense in X. We prove a strong converse: the likely intersections (correctly defined!) are dense in X in the Euclidean topology. This is a report on joint work with Sebastian Eterović.

Harry Schmidt (University of Basel)

Title: Effective isogeny estimates along families of abelian varieties

Abstract: The celebrated isogeny estimates of Masser and Wüstholz give an effective bound for the minimal degree of an isogeny between two abelian varieties A,B of dimension g. Fixing the dimension g (and assuming that A,B are defined over a number field), their bound is polynomial in the degree of the field of definition of A,B and the Faltings height of A. I am going to talk about joint work with Binyamini in which we prove an effective polynomial estimate for the minimal degree of an isogeny between an abelian variety A and a member B of a fixed family of abelian varieties. Our bound only depends on the Faltings height of A and the family but not on the particular member of the family. This has some direct applications to problems in unlikely intersections such as an effective and uniform version of a theorem of Orr.
Time permitting I will talk about Orr's theorem in more detail and motivate extensions to global fields of positive characteristic.

Margaret Thomas (Purdue University)

Title: Effective Pila--Wilkie bounds for Pfaffian sets and some diophantine applications

Abstract: Following critical insights of Pila and Zannier, there are by now a great many applications of o-minimality to diophantine geometry arising from the celebrated counting theorem of Pila and Wilkie, as well as from extensions of this result due to Pila and Habegger--Pila. The proof of the Pila--Wilkie Theorem (and that of its variants) does not, however, provide an effective bound. I will discuss some ongoing joint work in progress (with Gal Binyamini, Gareth O. Jones and Harry Schmidt) in which we obtain effective forms of the Pila--Wilkie Theorem and its variants for sets definable in various structures described by Pfaffian functions (including an effective Yomdin--Gromov parameterization result for sets defined using restricted Pfaffian functions), and then use these effective estimates to derive several diophantine applications, including an effective, uniform Manin--Mumford statement for products of elliptic curves with complex multiplication.

Umberto Zannier (SNS Pisa)

Title: Infinite orbits in elliptic surfaces, Ax Schanuel and ramification in the Legendre family

Abstract: Motivated by work of Cantat-Dujardin, we study orbits by translations in surfaces with two elliptic fibrations. We prove in particular that all orbits are infinite away from a proper Zariski-closed subset. Among the tools, beyond the Pila-Wilkie counting, are a new version of Ax-Schanuel by Bakker-Tsimerman and a(n old) theorem of Shioda on ramification of sections of the Legendre family. This is joint work with Corvaja and Tsimerman.